assemblies. To satisfy this condition, the dimensions of the assemblies must be nominally equal, but the initial gap width must be in the inverse ratio to their elastic moduli. Dadson has shown, however, that this latter condition on the initial width is not particularly critical, probably due to the fact that the initial gap is small compared to the gap at higher pressures.

Using these assumptions, which are very sound provided adequately precise assemblies can be constructed and idealized conditions met, one can write an expression for the effective areas A_P (and) B_P of the two assemblies in the form:

$$A_P = A_0 [1 + \lambda_A f(P)] \text{ and } B_P = B_0 [1 + \lambda_B f(P)] \quad (9)$$

where $f(P)$ is an unknown function of pressure to be determined experimentally. Because of the principle of completely similar distortion, λ_B and λ_A are related by $\lambda_B = k\lambda_A$ where the constant k is the ratio of the two Young's moduli E_A and E_B or, alternately, the shear moduli G_A and G_B. In equation form:

$$k = \frac{E_A}{E_B} \quad \text{or} \quad k = \frac{G_A}{G_B}. \quad (10)$$

This clearly implies the ratios of the two sets of moduli are equal or, in other words, that the Poisson ratios of the two materials are equal. Since the term $\lambda_A f(P)$ and $\lambda_B f(P)$ are very small, one can write

$$\frac{A_P}{B_P} = \frac{A_0}{B_0} [1 + (\lambda_A - \lambda_B) f(P)]$$

$$= \frac{A_0}{B_0} [1 + \lambda_A (1 - k) f(P)]. \quad (11)$$

Two balances can be compared experimentally with sensitivity of a few parts in 10^6. The ratio A_P/B_P is thus measurable to a high degree of accuracy, and the quantity $\lambda_A f(P)$ can be extracted from equation (11) provided k is known. The quantity $\lambda_A f(P)$ is precisely the desired change in effective area of system A. When this function is known, the free-piston gage A is calibrated at high pressures, and the change in area of system B or any other system is readily available. Since the sensitivity to errors or uncertainties in k becomes very high if $(1-k)$ is small, it is desirable to construct the two systems with highly different elastic moduli but with equal Poisson ratios and also to know accurately the elastic moduli of both materials. These restrictions, coupled with the strength requirements and the need for precise machining properties severely limit the possible materials usable in such a study. The first two metals used by Dadson and coworkers were steel and an aluminum bronze known as "hydurax". In later work they also used a tungsten alloy "GEC heavy metal" to provide a three-way intercomparison as a self-consistent check on the method. The elastic constants of these materials are given in table 2. Since errors in elastic constants give a first-order error in λ, it is essential to obtain the best values possible for the elastic constants of both materials. Dadson used static values for the shear moduli and ultrasonic values for Poisson ratios and gave detailed reasons for this decision.

TABLE 2. Elastic parameters for metals in similarity intercomparison

	Young's modulus (E) (dyn/cm²)	Modulus of ridgidity (G) (dyn/cm²)	Poisson's ratio
Steel (K9)	20.5×10^{11}	7.86×10^{11}	0.295
"Hydurax" aluminum bronze	14.3×10^{11}	5.45×10^{11}	0.333
GEC heavy metal–tungsten alloy	36.7×10^{11}	$13.5_5 \times 10^{11}$	0.286_5

It is significant to note that the functional variation of effective area with pressure is still preserved in the similarity method. This is the only method for which this functional relation is experimentally available above the pressure available to the differential mercury manometer. Although elastic theories suggest a linear variation of area with pressure, nonlinear effects might be expected to be associated with changes in viscosity along the length of the gap due to this pressure variation of the viscosity. Such nonlinear variations were actually observed by Dadson using liquid paraffin as a fluid. However, when light mineral oil or castor oil was used as a transmitting fluid, the data agreed with a linear relationship such that disagreement in effective area of less than one part in 10^5 was demonstrated at measured pressures up to two kilobars. This linear relationship was thus assumed in later analysis. A further significant and surprising result of Dadson's work was the dependence of the distortion constant of a single piston cylinder assembly on the transmitting fluid used even when the linear relationship existed. The values of λ from equation (11) were, for example, approximately ten percent higher when using light mineral oil than when using castor oil. The magnitude of the effect was approximately the same for two different sized piston assemblies. These results are in marked contrast to the earlier work mentioned above at lower precision and lower pressures wherein the effective area was reported as not depending on the viscosity of the fluid.

It will be noted from table 2 that the condition for the Poisson ratio used in a "similarity" pair of metals is not well satisfied when the aluminum bronze is used in conjunction with either of the other two metals. Assuming the functional relationship $f(P)$ in equation (9) is simply P, Dadson has given an analysis indicating how this discrepancy in the Poisson ratio can be accounted for using elastic theory. The use of elastic theory in determining a correction term here is in contrast to the approach of Zhokhovskii and others given above where elastic theory is used to evaluate the total change in effective area of the assembly. To obtain these correction terms, Dadson defines

$$k=\frac{G_A}{G_B}\neq\frac{E_A}{E_B}$$

and shows that correction terms

$$\Theta_A=\frac{3\sigma_A-1}{2E_A} \text{ and } \Theta_B=\frac{3\sigma_B-1}{2E_B} \tag{12}$$

must enter the determination of λ as indicated:

$$(1-k)\lambda_A=(\lambda_A-\lambda_B)_{\text{meas}}+(\Theta_B-k\Theta_A). \tag{13}$$

It is interesting to note that these correction terms are of the same form and of approximately the same magnitude as the total change in area of the controlled-clearance free-piston gage described above.

The extension of the similarity method to three metals allows three separate experimental combinations yielding two direct determinations of λ for each individual piston-cylinder assembly and one indirect determination. Since the purpose of the investigation is to establish a primary scale, a determination of the distortion coefficient for the steel assembly is the only important quantity. The similarity method thus gives three measurements of this quantity although the indirect determination does have some dependence on the other two. Four to six independent sets of measurements were made on each of two piston-cylinder assemblies of identical design but with different nominal areas. Similarity measurements were made on one assembly to 500 bar and on the other to 1200 bar although the calibrated assembly was capable of 3000 bar. Similar measurements were also made on a piston-cylinder assembly of a different design capable of pressures to 6000 bar. The total dispersion of the λ values determined for the two low-pressure assemblies was approximately four percent, the largest errors of which appeared to be associated with intercomparisons with the bronze assembly, which has an unfavorable Poisson ratio comparison. A brief error analysis indicated better reliability of selected data, and a final value of $4.0_6\times10^{-7}$/bar was reported for λ associated with the steel piston-cylinder assembly with an estimated accuracy of two percent. The 6000 bar assembly exhibited a value of $3.0_6\times10^{-7}$/bar due to its different construction.

Two additional internal checks were also available. The λ value for each assembly was independently determined; therefore, an intercomparison of any two steel assemblies so calibrated yields a direct measurement of the differences in the values of λ so determined. For the two lower pressure assemblies, the difference in λ determined by the similarity method agreed within 1.5 percent compared to a direct intercomparison measurement of one percent. A similar intercomparison of a high-pressure gage with the low-pressure assemblies over the range possibly gave agreement of approximately two percent in λ. All internal checks thus indicate precision of the order of two percent in λ as measured by this method. This implies the accuracy of approximately one part in 10^5 in effective area at one kbar and one part in 10^4 at 10 kbar provided A_0 is known this well. It was also noted that the variation of effective area with pressure was linear to the maximum value of 6 kbar.

Dadson and coworkers developed the flow method as an independent check on the changes of effective area with pressure as measured with the similarity method. The flow method is based on the concept that a change in effective area associated with pressure distortion could be related to a known difference in area between two piston-cylinder assemblies similarly constructed but of slightly different zero pressure effective areas. Since the fluid flow in the crevice between the piston and cylinder is very sensitive to the gap width, the flow rate can be used as an indicator of changes in effective area. The most severe problem is obtaining a simple relationship between the measured flow rate and the effective area. This difficulty is associated with the varying viscosity along the crevice as well as the changes in pressure profile and gap width along the gap length. Dadson assumed the pressure profile along the gap length to be similar in the two systems and the width of the gap at any point to be proportional to the pressure at the point. Further details will not be given since the method is not of primary significance. Measurements were made to 1500 bar and to this pressure agreement with the similarity method of approximately three percent in the determination of λ was obtained. The dispersion of the measured points was slightly larger than in the similarity method.

d. Summary

Following the above survey of the three major approaches used to evaluate or eliminate the elastic distortion error in a free-piston gage it is appropriate to evaluate the relative merits of the three approaches. The independent determination of the freezing pressure of mercury at 0 °C gives a very good intercomparison between techniques. The results reported for these measurements are given in table 3 of section 3 on fixed points. Although the error flags of all measurements before and including that of Newhall, et al. (1963) overlap, there is an obvious overconfidence in the evaluation of systematic errors or theoretical uncertainties in at least one of the last four measurements indicated in table 3. With current understanding, one can only suggest possible weak points in each measurement and indicate areas for improvement.

As pointed out above and as indicated by the error flags associated with Zhokhovskii's determination of the mercury freezing pressure, his theoretical approach is not sufficiently reliable to be competitive